

Seminar 3 ECON4930 Tuesday 8 March

1. Consider the following social optimisation problem having N hydropower plants, and M thermal power plants with variable cost functions $c_i(e_{it}^{Th})$ ($i=1,\dots,M$):

$$\text{Max} \sum_{t=1}^T \left[\int_{z=0}^{x_t} p_t(z) dz - \sum_{i=1}^M c_i(e_{it}^{Th}) \right]$$

s.t.

$$x_t = \sum_{j=i}^N e_{jt}^H + \sum_{i=1}^M e_{it}^{Th}$$

$$R_{jt} \leq R_{j,t-1} + w_{jt} - e_{jt}^H$$

$$R_{jt} \leq \bar{R}_j$$

$$e_{it}^{Th} \leq \bar{e}_i^{Th},$$

$$x_t, e_{it}^H, e_{jt}^{Th}, R_{jt} \geq 0, i = 1, \dots, M, j = 1, \dots, N, t = 1, \dots, T$$

$$T, R_{j0}, \bar{R}_j, \bar{e}_i^{Th} \text{ given, } R_{jT} \text{ free, } j = 1, \dots, N$$

- 1a) Derive the first-order conditions.
 1b) Discuss the role of the plants as peak-load plants and base-load plants (hint: read pp. 111-116 in Hydropower economics).
 1c) Extra question: Is there a Hveding's conjecture for a mix of hydro and thermal plants? (hint: look at the situation for $t=T$ and do backward induction, remember the problem with large reservoirs that may pass a price-increasing event)
2. Consider the following social optimisation problem, having hydropower generation and two more types of generating technology; conventional thermal (gas turbines, combined cycle gas turbines (CCGT), coal-fired), nuclear power, and intermittent power generation consisting of wind power, solar power and small-scale hydro with no reservoirs together with other run-of-the-river hydro installations:

$$\max \sum_{t=1}^T \left[\int_{z=0}^{x_t} p_t(z) dz - c_C(e_t^C) - c_N(e_t^N) \right]$$

subject to the constraints

$$x_t = e_t^H + e_t^C + e_t^N + e_t^I$$

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \bar{R}$$

$$e_t^C \leq \bar{e}^C$$

$$e_t^N \leq \bar{e}^N$$

$$e_t^I \leq \bar{e}^I$$

$$x_t, e_t^H, e_t^C, e_t^N \geq 0, t = 1, \dots, T$$

$$T, R_0, \bar{R}, \bar{e}^C, \bar{e}^N, \bar{e}^I \text{ given, } R_T \text{ free}$$

2a) Discuss the modelling of the intermittent technologies.

2b) Derive the first-order conditions.

2c) Derive the shadow price for increasing hydro reservoir capacity, and derive the shadow prices for increasing the generation of electricity using intermittent technology.

2d) Discuss implications of variable wind for the utilisation of the other technologies within a two-period framework, using as a benchmark equal amounts of intermittent power for the two periods and a situation of equal prices for the two periods. (Hint: look at the figures in the slides for Lecture 5 and modify)

2e) Explain the situation leading to no use of hydro in a period. How can such situations be used to discuss the role of Hydro as a battery?

2f) Discuss how pumped storage may influence the price formation. (Hint: the energy used to pump up water is less than the energy generated when releasing the water again)